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SPECIAL MODES OF THE RIECKE PHENOMENON

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The modes of thermoacoustic oscillations upon manifestation of the action of a hysteresis attractor when a change of the laminar and turbulent regimes periodically occurs, has been determined analytically.

Introduction. Small-amplitude thermoacoustic self-oscillations of oscillating (pulsating) combustion improve the economic and performance indices of combustion chambers of industrial units. However, as the thermal load increases, their amplitudes grow and become destructive. Serious complications and disturbances in operation are caused by pulsating combustion in blast-furnace stoves [1]. The results of investigations of this nonstationary regime by different authors are contradictory [2]. The same feature is also observed when the theoretical solutions [1] are compared to experimental investigations performed at a number of metallurgical plants.

The simplest device in which thermoacoustic oscillations are set up is the Riecke tube. Its phenomenon lies in excitation of self-oscillations by heat supply from an internal heat source located in the lower part of a vertical tube that is open at both ends on formation of direct motion in the tube.

The mechanisms of thermoacoustic oscillations proposed by the author and considered below enable one to mathematically model the modes of periodic motions in the Riecke tube. A complete qualitative coincidence of the results of the earlier experimental investigations and the theoretical solutions is observed.

It is common knowledge that negative resistances of the corresponding nature form self-oscillation mechanisms in different phenomena [3, 4]. The manifestation of phenomenological delay contributes to the extension of the range of nonstationary operation, increases the oscillation amplitude, and represents an independent mechanism of self-oscillations [4].

The conditions of appearance of negative hydraulic and thermal resistances that are caused by the supply of heat have been determined in [5]. The first resistance is the basic mechanism of the Riecke phenomenon of excitation of acoustic oscillations by supplying the heat of constant power W of the heat flux from an electric coil (as has already been noted) in the lower part of a vertical tube [6, 7]. As the air flow rate in the Riecke tube increases, the temperature of the air moving in it decreases since the value of W of the heat flux is constant, which decreases its viscosity. This is responsible for the reduction in the hydraulic loss along the length with growth in the flow rate Q_{t} in the laminar regime and for the formation of the descending branch $h_l(Q_{\text{t}})$ of the negative hydraulic resistance. As the power W decreases, the mechanism of negative hydroresistance is attenuated; the self-oscillations are maintained due to the delay τ of heat transfer at high-temperature gradients [4] and volume relaxation of the heat supply [6].

Negative thermal resistance is responsible for the excitation of self-oscillations for a variable (flow-rate-dependent) power W of the heat flux. Apart from the phenomenological delay of the process of combustion, negative thermal resistance contributes to the formation of the "singing"-flame mechanisms, which, under the corresponding conditions [5], can generate self-oscillations in the Riecke tube when a burner is used for supply of heat instead of the electric coil.

Since the mechanisms of the Riecke phenomenon [5, 6] of excitation of oscillations remained constant for W = const, theoretical descriptions of the Riecke phenomenon rarely coincided with experimental ones even qualitatively [6]. Despite the seemingly contradictory experimental investigations of the Riecke phenomenon [6], they are reliable and can be modeled mathematically but are very limited [6, 7].

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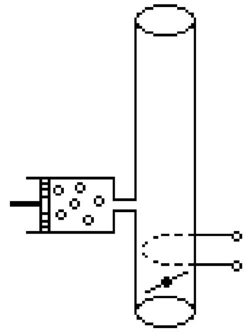

Fig. 1. Generator of thermoacoustic oscillations of the Riecke phenomenon.

Formulation of the Problem. Equations of nonstationary motion in the Riecke tube have been composed in [8]; the head characteristic allowing for the action of negative hydraulic or thermal resistance has been introduced into these equations. The developed algorithm of construction of periodic solutions of the above equations with the use of a modified Euler method of construction of integral curves [9] enables one to mathematically model any regularities and features of the Riecke phenomenon.

In the experiments of Lehmann [7], noteworthy is an unusual kind of oscillation mode in which a burst of pressure is periodically observed with increase in it. Such an oscillation mode, as follows from [10], is formed when a hysteresis attractor appears in the regime of intermittent turbulence.

This work seeks to mathematically model oscillation modes that are caused by the hysteresis attractor and are periodic solutions of the system of equations [8]

$$L_a \frac{dQ_t}{dt} = H(Q_t, W) - P\,, \quad C_a \frac{dP}{dt} = Q_t - \varphi(P)\,, \tag{1}$$

where $H(Q_t, W) = A(Q_t, W) - h_l(Q_t, W)$; inversion of the function $\varphi(P)$ is the dependence determining the scattering of the energy of entry and in the case of flow about the grid of the heater's coil: $h_g(Q_t, W) = kQ_t^2$; $P = p_o - p_t$ is the pressure difference (p_o is the atmospheric pressure and p_t is the pressure after the electric coil in the Riecke tube) and k is the hydraulic-loss factor.

Construction of Special Modes of the Riecke Phenomenon. If the equation of integral curves

$$\frac{dP}{dQ_t} = \frac{Q_t - \varphi(P)}{H(Q_t, W) - P}\,\frac{L_a}{C_a} \tag{2}$$

has a limiting cycle, the corresponding solution of system (1) is periodic. The oscillation modes $P(t)$ and $Q_t(t)$ are determined by integration of Eqs. (1) along the limiting cycle. Thermoacoustic oscillations are modeled in a generator (Fig. 1), where a change in its acoustic compliance C_a does not influence the hydraulic characteristics of the vertical tube.

The head characteristic $H(Q_t, W)$ of the Riecke tube, i.e., the generator of thermoacoustic oscillations (Fig. 1), on passage to the region of turbulent regime, as far as the flow rate Q_t is concerned, is linearly extended by the dashed line (Fig. 2), which reflects the hysteresis of the laminar regime with an instantaneous transition to the characteristic $H(Q_t, W)$ of turbulent flow. Change of the laminar and turbulent regimes is periodic [10] and is allowed for in construction of the limiting cycles and oscillation modes presented in Fig. 2 for a tube of length 10 m and diameter $d = 0.1$ m with $W = 15$ kW.

With decrease in the wave resistance Z, which is attained by increasing the volume changed (see Fig. 1), we have a change in the cycles and modes shown in Fig. 2. For a certain Z^*, the limiting cycles become constant $\forall Z$: $Z < Z^*$ and consisting of the horizontal lines and portions of the characteristic $H(Q_t, W)$ [9]. A necessary condition of their formation is the saddle form of the head characteristic, whereas a sufficient condition is the presence of a considerable volume of the accumulating capacity of the oscillatory circuit of the Riecke tube. Upon the manifestation of the hysteresis attractor, the constant limiting cycles remain the same for any flow rates Q_t corresponding to the ascend-

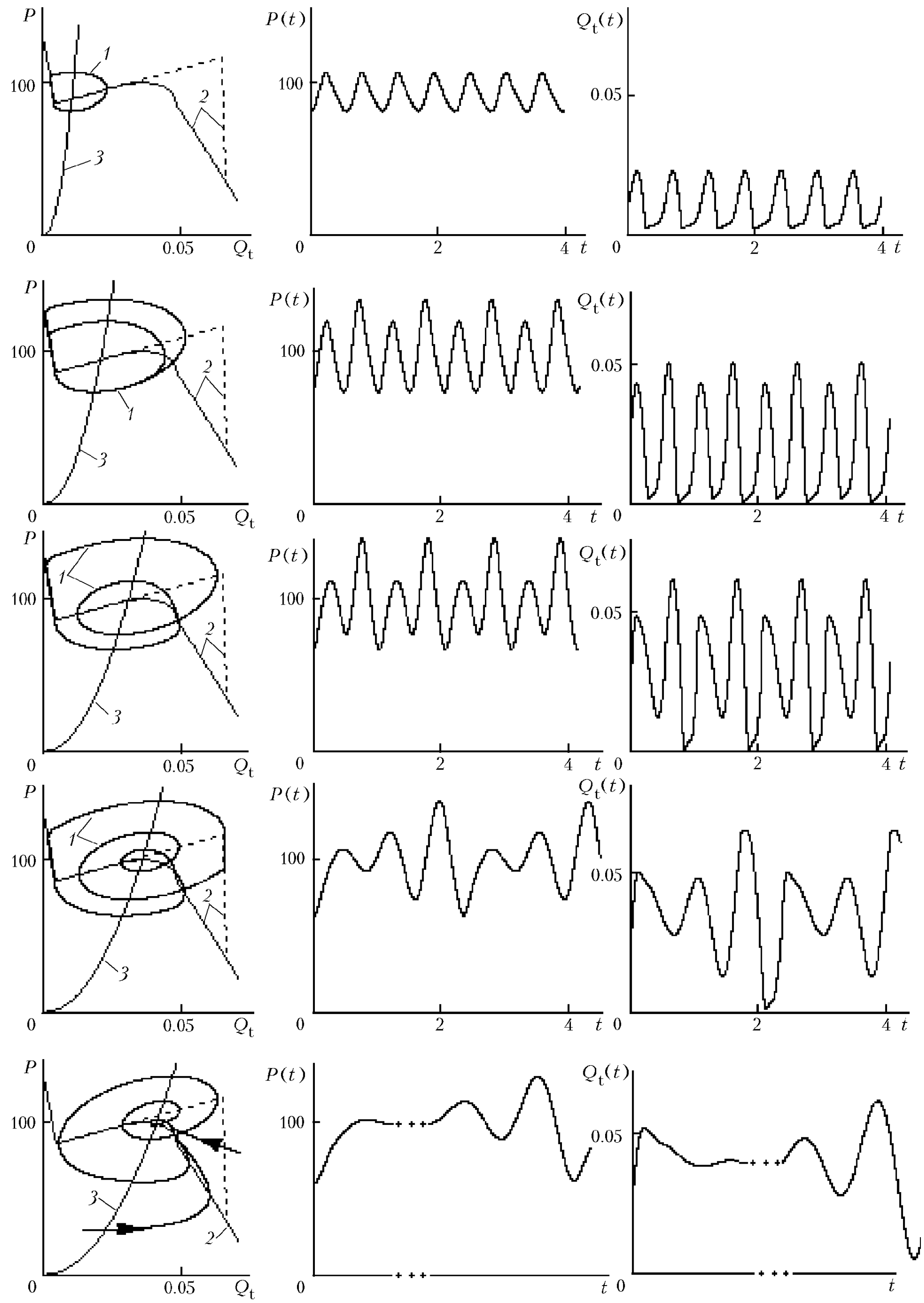

Fig. 2. Limiting cycles (1) and the corresponding modes of the oscillations $P(t)$, Pa, and $Q_t(t)$, m^3/sec, for the wave resistance $Z = 10^3$ Pa/(m^3·sec) when the air flow rate is changed by throttling at entry; 2) head characteristic of the Riecke tube; 3) characteristic of the grid in front of its oscillatory circuit; dashed sloping line, hysteresis of the laminar regime, dashed vertical line, its transition to a turbulent regime, symbols, infinite approximation of the integral curve to a stationary regime.

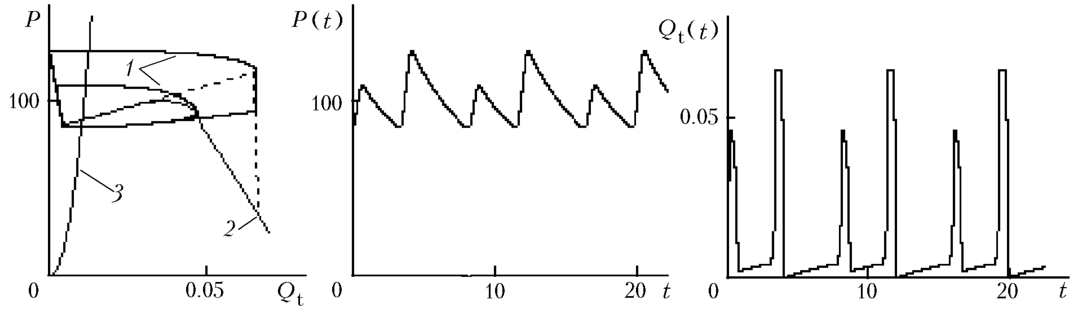

Fig. 3. Limiting cycles (1) due to the hysteresis attractor and the mode of the oscillations $P(t)$, Pa, and $Q_t(t)$, m^3/sec, for the wave resistance $Z = 3.1 \cdot 10^2$ Pa/(m^3/sec) $\approx Z^*$, the heat-flux power $W = 15$ kW, and an air flow rate of 0.01 m^3/sec, 2) head characteristic of the Riecke tube, 3) characteristic of the grid in front of its oscillatory circuit.

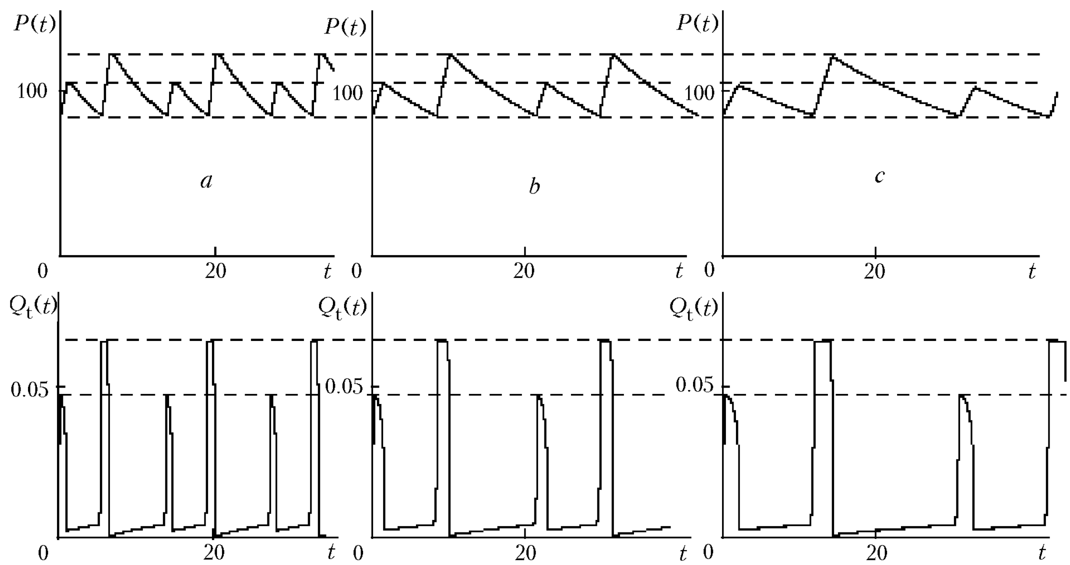

Fig. 4. Character of change in the relaxation self-oscillations $P(t)$, Pa, and $Q_t(t)$, m^3/sec, upon manifestation of a hysteresis attractor with decrease in the quantities $Z < Z^*$: a) $Z = 2.2 \cdot 10^2$; b) $1.7 \cdot 10^2$; c) $1.4 \cdot 10^2$.

ing branch of the head characteristic $H(Q_t, W)$ but, at its ends, the larger limiting cycle becomes unstable, whereas the smaller one degenerates into a stable node or focus.

The constant limiting cycles and the corresponding mode of relaxation oscillations for $Z \leq Z^*$ are shown in Fig. 3.

With further decrease in the values $Z < Z^*$, the amplitudes (accordingly the larger and the smaller (Fig. 3)) remain constant; only the mode changes in connection with the increase in the period (see Fig. 4).

Conclusions. The constancy of the oscillation amplitude when the condition $\forall Z$: $Z < Z^*$ is satisfied occurs in the absence of the manifestation of the attractor, too.

A constant limiting cycle is also formed in pneumatic systems and hydrosystems for any vane blower even with an ascending branch of the head characteristic but on condition of the presence of a large accumulating capacity in the system. Such a head characteristic becomes saddle-type because of the presence of its branch in the region of negative flow rates.

Excitation of constant-amplitude relaxation oscillations in systems involving a vane blower is also determined by the conditions considered above, since the equations of the theory of Riecke phenomenon [8] and the theory of surging of vane blowers [11] formally coincide.

The reason for the formation of an unstable ascending branch of the head characteristic $H(Q)$ of centrifugal blowers is also the negative (but vortex) hydraulic resistance caused by the generation of separating flows and vortex formation with reduction in the supply in its diffusor channels.

NOTATION

$A(Q_t, W)$, thrust of the Riecke tube, Pa; C_a, acoustic compliance, $m^3 \cdot sec^{-1}$/(Pa/sec); d, tube diameter, m; $H(Q_t, W)$, head characteristic of the Riecke tube, Pa; $h_l(Q_t, W)$, characteristic of the grid in front of the oscillatory circuit in the Riecke tube, Pa; $h_l(Q_t, W)$, hydraulic loss along the tube length, Pa; L_a, acoustic mass, Pa·sec/(m^3/sec); l, tube length; P, rarefaction after the heater in the Riecke tube, Pa; p, total pressure in the cross section of the flow, Pa; Q_t, volume flow rate of the heated air, m^3/sec; t, running time, sec; W, heat flux, kW; Z, wave resistance, Pa/(m^3/sec); τ, delay time. Subscripts: a, acoustic quantity; e, atmospheric (environmental); t, value of the parameter in the heated medium; *, wave resistance for which a constant limiting cycle is formed; g, grid.

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